GENERAL DESIGN OF CRITICALLY DAMPED GALVANOMETERS

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CONTENTS

I. INTRODUCTION

The theory of the galvanometer, when used under the critically damped condition as a detector of constant voltage in a circuit of fairly low resistance, is given in papers by White [1] and Jaeger [2], published about 10 years ago. In these papers some of the equations which we shall use are given. However, we wish to consider sensitivities to current and voltage impulses as well as to steady current and voltage, and in order that the matter may be presented as a unit, the theory, in so far as it is needed in the design of such galvanometers, will be given here. That is, we shall show the relations between those constants of a galvanometer with which the user is concerned and those constants with which the maker is concerned.

[1] Physical Review, **19**, p. 305; 1904.

[2] Zeitschrift für Instrumentenkunde, **23**, p. 261 and p. 353, 1903; and Annalen der Physik, **326**, p. 64, 1906.

The user is concerned with the period, the resistance which must be put in series or parallel in order to give critical damping, and the sensitivity to the quantity the galvanometer is used to detect or measure. These will be referred to as the operation constants. The maker is concerned with the inertia constant, the damping constant, the restoring constant, the dynamic constant, and the resistance—constants which depend upon the kind of material used, the number of turns, the intensity of magnetization, and the size and the proportions of the parts of the galvanometer. These will be referred to as the intrinsic or construction constants. These two sets of constants are necessarily interdependent, and a knowledge of the relations existing between them is necessary for an understanding of the subject of galvanometer design. We shall, therefore, show in what manner each of the operation constants depends upon the intrinsic constants.

These relations will then be used in establishing a procedure for finding a set or sets of values for the intrinsic or construction constants, such as will give previously selected or specified values for those of the operation constants which pertain to the class of work in which the galvanometer is to be used. The finding of some set of values for the intrinsic constants which may be realized in the construction without unnecessary difficulty, and which will give the specified values for the operation constants, constitutes what we call the general design of a galvanometer. The matter will be considered primarily from the standpoint of the design of moving coil galvanometers of high sensitivity, though much of the discussion will apply equally well to the less sensitive galvanometers with pointers and to galvanometers of the moving magnet type. The way in which particular values for the intrinsic constants may be realized, and other matters pertaining to what we call the detail design, can not be considered in this paper.

II. THEORY

1. THE OPERATION CONSTANTS

Galvanometers are used critically damped in four distinct classes of measurements, in each of which the quantity to be detected or measured is different.

In the first (which we shall refer to as class A) it is the current, or change in the current, in a circuit including the galvanometer. The measurement of insulation resistance by the direct deflection method using a fairly high voltage is an example of this class of

measurements. Here the sensitivity which should be considered is to current.

In the second (or class B) it is the change in the voltage in the circuit in which the galvanometer is connected which is to be detected or measured. The change in voltage in the galvanometer circuit, on reversing the test current through a bridge which is not exactly balanced, is an example of this class of measurements. Here the sensitivity which should be considered is to voltage.[3]

In the third (or class C) it is the quantity of electricity suddenly passed through the circuit, including the galvanometer, which is to be detected or measured. The comparison of the capacities of condensers by the ballistic throws given, when they are charged to the same voltage and discharged through the same galvanometer, and the comparison of capacities by the method of mixtures, are examples of this class of measurements. Here the sensitivity which should be considered is to impulsive rush of a quantity of electricity $q=\int idt$ through the circuit including the galvanometer.

In the fourth (or class D) it is the time integral of the electromotive force in the circuit. Examples of this class of measurements are the comparison of magnetic fields by the ballistic throws they give, when a coil connected in series with the galvanometer is suddenly removed first from one field and then from the other, or the comparison of two mutual inductances by connecting their secondaries differentially in series with the galvanometer and simultaneously reversing the currents in their primaries, adjusting the ratio of the currents so as to make the ballistic throw zero. Here the sensitivity which should be considered is to the $\int edt$, where e (the voltage) is negligibly small except for a very short time.

Besides the sensitivity to the quantity to be detected or measured the user of a galvanometer is concerned with the time between the change which causes the deflection and the instant when the deflection may be considered to have reached a constant value; or, in the case of an impulse, the instant when the deflection has reached a maximum value and must be read. We shall refer to the first as the deflection period and the second as the ballistic period.

[3] Under any definite conditions the voltage sensitivity is equal to the current sensitivity divided by the resistance. It, therefore, is not necessary to introduce the idea of a voltage sensitivity as distinct from the current sensitivity. However, for those cases in which it is the voltage rather than the current which is the quantity to be measured or which is the independent variable, the operation of the galvanometer is much more readily understood if we refer to its voltage sensitivity rather than to its current sensitivity.

In all cases the user of a galvanometer is concerned with the conditions which give critical damping; that is, the conditions which make the motion of the moving system just aperiodic (or just deadbeat). The damping often depends to a very marked degree upon the resistance of the circuit into which the galvanometer is connected. The resistance of the circuit (not including the resistance of the galvanometer) in which the motion of the moving system is critically damped will be referred to as the external critical resistance. In most cases it is an important operation constant of the galvanometer. If the apparatus with which the galvanometer is being used has a resistance between the terminals to which the galvanometer is connected different from the external critical resistance, critical damping can be brought about with most galvanometers of the moving coil type and with some of the moving magnet type. In some cases it is done by putting resistance in series with the apparatus and galvanometer, and in other cases resistance is put in parallel across the terminals to which the galvanometer is connected.

Sometimes it is desired that the galvanometer be approximately critically damped without special adjustment for all readings or settings of the apparatus with which it is used, when the resistance of the apparatus between the terminals to which the galvanometer is connected varies with the setting through wide limits. This condition is usually easily brought about by the use of two resistances, one connected in parallel and the other connected in series with the galvanometer. By a proper choice of the resistances, the damping can be made very nearly critical regardless of the resistance of the apparatus with which the galvanometer is used, but to do so necessarily reduces the sensitivity very materially. It is also possible to design a galvanometer so that, without a shunt or parallel resistance, it will be approximately critically damped when used in circuits of almost any resistance. This is accomplished by the use of suitable air dampers or by the use of an auxiliary winding closed upon itself. Sometimes the winding of a moving coil galvanometer is placed on a metal frame which serves also as the auxiliary closed circuit winding. The use of an auxiliary winding makes the construction of a galvanometer to have a high sensitivity to voltage or voltage impulse and a short deflection period much more difficult.

For general laboratory use it is desirable that the rate of change of the damping (or, more specifically, the rate of change of the deflection period) with the change in resistance of the external

circuit shall be small. Its value, or something equivalent to it, might be considered as one of the operation constants. However, where a high or fairly high sensitivity is required it is better to make the adjustments necessary to give critical or approximately critical damping. Where this is done, the rate of change of the damping with change of the external resistance is of but little importance.

The operation constants with which the user of a galvanometer may be concerned, and the symbols here used to represent them, are as follows:

R = the external critical resistance,
T_d = the deflection period,
T_b = the ballistic period,
S_i = the current sensitivity,
S_e = the sensitivity to voltage in a circuit having a resistance giving critical damping,
S'_e = the sensitivity to voltage in a circuit of resistance exceeding the critical resistance,
S_q = the sensitivity to quantity or $\int idt$,
S_n = the sensitivity to $\int edt$ in a circuit having a resistance giving critical damping, and
S'_n = the sensitivity to $\int edt$ in a circuit having a resistance in excess of the critical resistance.

For any one of the four classes of measurements we need consider only the appropriate period and sensitivity, and usually the resistance of the apparatus between the terminals to which the galvanometer is connected, or the external critical resistance of the galvanometer.

2. THE INTRINSIC CONSTANTS

As has been pointed out above, the values of the operation constants necessarily depend upon the size, shape, and arrangement of the parts of the galvanometer; the kind of material used in the construction; the intensity of magnetization, etc. That is, they depend upon the values of the intrinsic or construction constants. The intrinsic constants and the symbols here used to represent them are as follows:

K = the inertia constant (the moment of inertia),
D = the damping constant (the ratio of the drag or retarding torque on the moving system to its rate of displacement, with the circuit open),

U = the restoring constant (the ratio of the restoring torque to the displacement),

G = the dynamic or displacing constant (the ratio of the displacing torque to the current), and

R_g = the resistance of the galvanometer.

Other symbols used and the quantities which they represent are as follows:

e = the voltage impressed in the circuit in which the galvanometer is connected,

i = the current in the circuit in which the galvanometer is connected,

θ = the angular displacement of the moving system,

t = the time,

s = the resistance of the shunt across the galvanometer terminals,

r = the resistance in series with the galvanometer,

R' = the resistance in series with the galvanometer if in excess of the external critical resistance,

r' = the resistance of an auxiliary closed winding,

g = the dynamic constant of an auxiliary closed winding,

D' = the damping constant with the auxiliary winding open, and

j, l, m, n, and p = constants defined by equations (66), (72), (49), (50), and (83).

3. OPERATION CONSTANTS IN TERMS OF INTRINSIC CONSTANTS

The equation generally accepted as representing the motion of the moving system of a galvanometer [4] is

$$K\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+U\theta=Gi \tag{1}$$

where i is the current in the winding or coil. This equation, however, is not in a convenient form, since as galvanometers are generally used the current, i, is affected by the motion of the moving system and so is not known. The current may, nevertheless, be expressed in terms of the impressed voltage, that is, the voltage having its seat in the apparatus with which the galvanometer is used, the voltage generated by the motion of the moving system and the constants of the circuit.

As is well known, especially to those who are familiar with the principles of operation of dynamo-electric machinery, that part of the electrical power supplied to the galvanometer which is converted into mechanical power is i times the generated or back

[4] Gray, Absolute Measurements in Electricity and Magnetism, 2, Part II, p. 392.

voltage. Also the mechanical power is the torque, Gi, times the angular velocity, $\frac{d\theta}{dt}$. Since these two quantities must be equal it follows, therefore, that the generated voltage is $-G\frac{d\theta}{dt}$. Consequently, if $r+R_g$ is the resistance of the circuit and e is the impressed voltage,

$$i=\frac{e}{r+R_g}-\frac{G}{r+R_g}\frac{d\theta}{dt} \tag{2}$$

This value of i substituted in equation (1) gives as the equation of motion of the moving system

$$K\frac{d^2\theta}{dt^2}+\left(D+\frac{G^2}{r+R_g}\right)\frac{d\theta}{dt}+U\theta=\frac{Ge}{r+R_g} \tag{3}$$

If, for any reason, the galvanometer is shunted by a resistance, s, the current which would flow through the coil in case the impressed voltage only were effective, is $\frac{es}{rs+rR_g+sR_g}$ and in case the generated voltage only were effective, is $-\frac{G(s+r)}{rs+rR_g+sR_g}\frac{d\theta}{dt}$. Where R_g is the resistance of the galvanometer and r is the resistance of the apparatus with which it is used, or which is in series with the galvanometer, and in which there is the impressed voltage, e. Since the current through the coil is equal to that which would flow as a result of the impressed voltage only, plus that which would flow as a result of the generated voltage only, it follows that for this case the equation of motion is

$$K\frac{d^2\theta}{dt^2}+\left(D+\frac{G^2(s+r)}{rs+rR_g+sR_g}\right)\frac{d\theta}{dt}+U\theta=\frac{Gse}{rs+rR_g+sR_g} \tag{4}$$

These may be considered the general equations for the motion of the moving systems of galvanometers, excepting in those cases in which it is necessary to take into consideration either the effects of self or mutual inductance or of capacity. That the relations are at least approximately correct has been verified by numerous checks between values for constants found by using these relations and those values found by other means. It must not be presumed, however, that they are correct to a high order of accuracy, and it is not necessary for the purpose of this paper that they should be.

Equations of the type of (3) and (4) are considered in elementary textbooks on differential equations and in their solution there is obtained the auxiliary equation

$$a^2f+ag+h=0$$

in which f, g, and h are the coefficients of $\frac{d^2\theta}{dt^2}, \frac{d\theta}{dt}$ and θ. The form of the solution will depend upon whether

$$g^2 > 4fh, \; g^2 = 4fh \text{ or } g^2 < 4fh$$

as well as upon what function e is of t. When some change is made in the apparatus with which the galvanometer is used so that e suddenly assumes a new constant value, or when the moving system of the galvanometer is released from a deflected position, the moving system takes up the new position in the minimum time (and the motion is said to be just aperiodic or critically damped) if the two roots of the auxiliary equation just referred to are equal. That is, if

$$\frac{G^2}{r+R_g} + D = 2\sqrt{KU} \tag{5}$$

or if

$$\frac{G^2(s+r)}{rs+rR_g+sR_g} + D = 2\sqrt{KU} \tag{6}$$

The particular value of r which satisfies equation (5) is the external critical resistance of the galvanometer and is designated as R. Therefore it follows from equation (5) that

$$R = \frac{G^2}{2\sqrt{KU}-D} - R_g \tag{7}$$

If the resistance of the apparatus with which the galvanometer is used (measured between the terminals to which the galvanometer is connected) is less than the external critical resistance, then critical damping may be brought about by connecting the proper amount of resistance in series with the galvanometer. If the resistance of the apparatus with which the galvanometer is used is more than the external critical resistance of the galvanometer, critical damping may usually be brought about by connecting a resistance, s, of suitable value in parallel with the galvanometer. In this case the value of s must be chosen so that

$$sR'/(s+R') = R \tag{7a}$$

where R' represents the resistance of the apparatus.

If the galvanometer is critically damped without the use of a parallel resistance, it follows from equations (3), (5), and (7), that

$$K\frac{d^2\theta}{dt^2} + 2\sqrt{KU}\frac{d\theta}{dt} + U\theta = \frac{(2\sqrt{KU}-D)e}{G} \tag{8}$$

If a parallel resistance is used to bring about critical damping, it follows from equations (4), (6), (7), and (7a) that

$$K\frac{d^2\theta}{dt^2}+2\sqrt{KU}\frac{d\theta}{dt}+U\theta=\frac{(G^2-2R_g\sqrt{KU}+DR_g)e}{R'G} \tag{9}$$

and if conditions are such that the galvanometer has no appreciable effect upon the magnitude of the current, i, in the main circuit, then

$$K\frac{d^2\theta}{dt^2}+2\sqrt{KU}\frac{d\theta}{dt}+U\theta=\frac{(G^2-2R_g\sqrt{KU}+DR_g)i}{G} \tag{10}$$

Here, since the galvanometer is shunted, i is not the current in its winding or coil.

Equations (8), (9), and (10) are the general equations for the motion of the moving systems of critically damped galvanometers and are the equations on which the work which follows is based. Equation (8) applies in case critical damping is brought about without the use of a shunt, and the impressed voltage is the independent variable. Equation (10) applies in case the galvanometer is shunted to bring about critical damping, and the current may be considered the independent variable. Equation (9) applies in case it is necessary to use a shunt to bring about critical damping and when the impressed voltage, rather than the current, must be considered the independent variable. Equation (9) applies in the cases which are intermediate between those to which equation (8) applies and those to which equation (10) applies.

Following a change in the value of the voltage or current from one steady value to another, there results a change in the steady deflection. If the change in voltage is Δe, or current is Δi, and the resulting change in the steady deflection is $\Delta\theta$, the ratio of $\Delta\theta$ to Δe, or to Δi, is the sensitivity. Since, when the deflection becomes steady, both $\frac{d^2\theta}{dt^2}$ and $\frac{d\theta}{dt}$ are zero, inspection of equations (8), (9), and (10) shows that

$$S_e=\frac{2\sqrt{KU}-D}{GU} \tag{11}$$

$$S'_e=\frac{G^2-2R_g\sqrt{KU}+DR_g}{R'GU} \tag{12}$$

$$S_i=\frac{G^2-2R_g\sqrt{KU}+DR_g}{GU} \tag{13}$$

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where S_i is the sensitivity of the galvanometer to current when connected in a circuit of high resistance, S_c is the sensitivity of the galvanometer to voltage having its seat in apparatus of resistance equal to the external critical resistance of the galvanometer, and S'_e is the sensitivity of the galvanometer to voltage in apparatus having a resistance R' in excess of the external critical resistance of the galvanometer, so that the galvanometer must be shunted to bring about critical damping.

If, instead of the voltage or current assuming a new constant value, it assumes a fairly large value for a very short time, after which it becomes zero or assumes its former value, an impulse is given to the moving system. If the time of the impulse is very short in comparison with the time of throw, during the impulse the second and third terms of the left-hand members of equations (8), (9), and (10) are very small in comparison with the first, so may be neglected. Then by a single integration over the time of the impulse it follows that

$$K\frac{d\theta}{dt}=\frac{2\sqrt{KU}-D}{G}\int edt \tag{14}$$

$$K\frac{d\theta}{dt}=\frac{G^2-2R_g\sqrt{KU}+DR_g}{R'G}\int edt \tag{15}$$

and

$$K\frac{d\theta}{dt}=\frac{G^2-2R_g\sqrt{KU}+DR_g}{G}\int idt \tag{16}$$

While here no consideration is given to self-induced voltage, which may during a part of the impulse be of the same order of magnitude as e, it may easily be shown that no appreciable error is introduced on this account, unless the self-inductance is so large that the electrical time constant of the circuit is appreciable in comparison with the time of the throw of the moving system.

After the impulse the right-hand members of equations (8), (9), and (10) are zero, so that for each of the three cases the equation of the motion of the moving system is:

$$K\frac{d^2\theta}{dt^2}+2\sqrt{KU}\frac{d\theta}{dt}+U\theta=0 \tag{17}$$

The solution of this equation (which is a special form of equations (3) and (4) considered above) is as follows:

$$\theta=C_1\epsilon^{-\sqrt{U/K}\,t}+C_2t\epsilon^{-\sqrt{U/K}\,t} \tag{18}$$

where C_1 and C_2 are constants of integration and ϵ is the base of the Naperian logarithm. Here C_1 is zero, since at the end of the

impulse both θ and t may be considered zero. Differentiation of equation (18) gives

$$\frac{d\theta}{dt} = -C_2 t\sqrt{U/K}\epsilon^{-\sqrt{U/K}t} + C_2\epsilon^{-\sqrt{U/K}t} \tag{19}$$

from which it follows that θ is a maximum when $t = \sqrt{K/U}$ or that

$$T_b = \sqrt{K/U} \tag{20}$$

where T_b is the time of throw [5] or the ballistic period. It also follows that C_2 is equal to the value of $\frac{d\theta}{dt}$ when $t = 0$, which is the value of $\frac{d\theta}{dt}$ as given by equation (14), (15), and (16). The substitution of these values for C_2 and t in equation (18) gives

$$S_n = \frac{\Theta}{\int edt} = \frac{1}{\epsilon G}\left[2 - \frac{D}{KU}\right] \tag{21}$$

$$S'_n = \frac{\Theta}{\int edt} = \frac{G^2 - 2R_g\sqrt{KU} + DR_g}{\epsilon R'G\sqrt{KU}} \tag{22}$$

$$S_q = \frac{\Theta}{\int idt} = \frac{G^2 - 2R_g\sqrt{KU} + DR_g}{\epsilon G\sqrt{KU}} \tag{23}$$

where Θ is the maximum value of θ or the magnitude of the ballistic throw, and S_n, S'_n, and S_q are the sensitivities to voltage and current impulses.

From equation (18) it will be seen that the deflection reaches a maximum and then becomes zero after a long time. However, when $t = (2\pi + 1)\sqrt{K/U}$ the deflection has passed and is then only a little more than 1 per cent of its maximum value. Since, in returning from the deflected position, the motion follows the same law as when e is suddenly changed from one constant value to another, the system accomplishes nearly 0.99 part of its final steady deflection in a time equal to $2\pi\sqrt{K/U}$. For most purposes one is not concerned in reading deflections to as close as 1 per cent, so we shall consider that

$$T_d = 2\pi\sqrt{K/U} \tag{24}$$

where T_d is the deflection period. That is, we shall consider the deflection period to be the same as the complete undamped period.

[5] The effect of damping, especially critical damping, upon the time of throw or ballistic period and upon the ballistic sensitivity is discussed by Stewart, Physical Review, **16**, p. 158, 1902. The effect of the damping by the current in a circuit of low resistance resulting from the voltage generated by the motion of the moving system is discussed by Jones, Proceedings of the Physical Society, **26**, p. 75, 1914.

Collecting the equations representing the relations[6] between operation and intrinsic constants gives

$$R=\frac{G^2-R_g(2\sqrt{KU}-D)}{2\sqrt{KU}-D} \tag{25}$$

$$T_d=2\pi\sqrt{K/U} \tag{26}$$

$$T_b=\sqrt{K/U} \tag{27}$$

$$S_e=\frac{2\sqrt{KU}-D}{GU} \tag{28}$$

$$S'_e=\frac{G^2-R_g(2\sqrt{KU}-D)}{R'GU} \tag{29}$$

$$S_i=\frac{G^2-R_g(2\sqrt{KU}-D)}{GU} \tag{30}$$

$$S_n=\frac{1}{\epsilon G}\left[2-\frac{D}{\sqrt{KU}}\right] \tag{31}$$

$$S'_n=\frac{G^2-R_g(2\sqrt{KU}-D)}{\epsilon G\sqrt{KU}\,R'} \tag{32}$$

$$S_q=\frac{G^2-R_g(2\sqrt{KU}-D)}{\epsilon G\sqrt{KU}} \tag{33}$$

It may be of interest to note that in case D is very small in comparison with $\sqrt{KU}$

$$S_e=\frac{2}{G}\sqrt{\frac{K}{U}} \tag{34}$$

and

$$S_n=\frac{2}{\epsilon G} \tag{35}$$

and in case $D=2\sqrt{KU}$

$$S_i=\frac{G}{U} \tag{36}$$

$$S_q=\frac{G}{\epsilon\sqrt{KU}} \tag{37}$$

that is, the relations are much simpler than in the corresponding equations above.

[6] It should be understood that the magnitudes of the operation constants are to be expressed in the system of units in which the intrinsic constants are expressed and not in the units usually employed.

4. RELATIONS BETWEEN OPERATION CONSTANTS

The fact that a galvanometer has but five intrinsic constants is evidence that all of the relations given by equations (25) to (33) can not be independent and that there must be some relations between the operation constants. An inspection will show that there is a simple relation between the ballistic and deflection periods, and between the four sensitivities. Expressing the other operation constants in terms of R, T_d, and S gives

$$T_b = \frac{T_d}{2\pi} \tag{38}$$

$$S'_e = \frac{R}{R'} S_e \tag{39}$$

$$S_i = R S_e \tag{40}$$

$$S_n = \frac{2\pi S_e}{\epsilon T_d} \tag{41}$$

$$S'_n = \frac{2\pi R S_e}{R' T_d \epsilon} = \frac{R}{R'} S_n \tag{42}$$

$$S_q = \frac{2\pi R S_e}{\epsilon T_d} = R S_n \tag{43}$$

These equations show what should be expected from a galvanometer intended for use in class B work, if used with apparatus having resistance between galvanometer terminals in excess of the external critical resistance of the galvanometer, or when used in class A, C, or D work.

The way the sensitivity changes as the resistance of the apparatus (between galvanometer terminals) increases, keeping the damping critical either by resistance in series or in parallel, is shown in Fig. 1. The point where the direction of the curves changes abruptly is where it is necessary to change the resistance from series to parallel to keep the damping critical, or where the resistance of the apparatus is equal to the external critical resistance of the galvanometer.

III. GENERAL DESIGN

1. NUMBER OF OPERATION AND INTRINSIC CONSTANTS

Equations (25) to (33) give the relations between the operation and intrinsic constants; that is, they enable one to calculate the values of the operation constants for known values of the intrinsic constants. But in the general design of galvanometers

the problem is to find values or sets of values for the intrinsic constants such as will give specified values for the operation[7] constants. An inspection of the equations shows that as they stand it would be difficult to use them for this purpose. For example, consider the design of a galvanometer for use with a bridge having a specified resistance between the terminals to which the galvanometer is to be connected, to give a specified deflection per unit of voltage which would be between the galvanometer terminals of the bridge with the galvanometer circuit open, and to

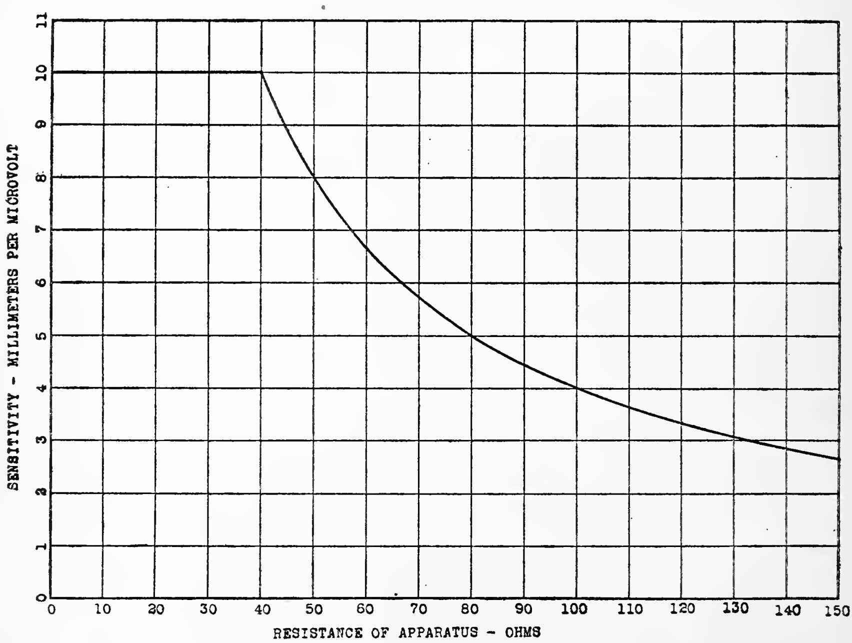

FIG. 1.—*Curve showing how the sensitivity to voltage changes as the resistance of the apparatus, with which the galvanometer is used, is changed. The curve applies for a galvanometer having an external critical resistance of 40 ohms and a sensitivity of 10 millimeters per microvolt*

come to rest in a deflected or zero position in specified time. The problem, then, is to find a set of values for the intrinsic constants such as will give the specified values for R, T_d, and S_e.

[7] The values of the five intrinsic constants of a galvanometer may be determined from measurements of five independent operation constants, or if one or more of the intrinsic constants is measured directly, then the remaining intrinsic constants may be determined from measurements of four or fewer independent operation constants. The equations representing the relations between the intrinsic constants and the measured operation constants may be used for calculating the values of the intrinsic constants necessary to give previously selected values for operation constants. For example, we have the equations given in this Bulletin, 6, p. 361, 1910. These could be used for determining values for the inertia constant, the damping constant, the restoring constant, and the dynamic constant necessary to give previously selected values for the alternating-current sensitivity, the alternating-voltage sensitivity, the direct-current sensitivity, and the resonating frequency; the value for the resistance of the galvanometer being chosen arbitrarily.

The relations between the operation constants and the intrinsic constants are given by equations (25), (26), and (28). However, it will be seen that one can not readily choose a set of values such that when they are substituted in these equations they give the value specified for each of the three operation constants. Even if he should find a set of values which would give the specified values for the operation constants, they would probably be such as would make the detail design and construction unnecessarily difficult. It is therefore desirable that the matter be investigated for the purpose of establishing, if possible, a definite procedure for finding values for the construction constants of galvanometers such as will give specified values for the operation constants, pertaining to each of the four different classes of work considered above. It is also important that the procedure enable us to know definitely the limits to the values for each of the intrinsic constants and something of the relations between them.

It has been shown above that there are only three independent relations expressed by equations (25) to (33). It has also been pointed out that for any class of measurements the user of a galvanometer is concerned, at most, with only three operation constants, while the galvanometer has five intrinsic constants. This suggests that possibly values for two or more of the intrinsic constants may be chosen arbitrarily [8] or within limits and values then calculated for the others such as will give specified values for the operation constants.

2. INTRINSIC CONSTANTS IN TERMS OF OPERATION CONSTANTS

An inspection of equations (25) to (33) shows that all can be satisfied with a zero value for both the resistance, R_g, and the damping constant, D. But the construction of a galvanometer for which either R_g or D is zero is impossible, though in some cases either or both may be made so small as to have no appreciable effect. If, however, the galvanometer is to have a high sensitivity to voltage or voltage impulse in a circuit of low resistance, then neither can conveniently be made so small that its effect may be neglected.

Considering the external critical resistance R, the deflection period T_d, the sensitivity S_e, the resistance R_g, and the damping constant D as fixed, a solution of equations (25), (26), and (28)

[8] A consideration of the matter from the standpoint of the power available for producing a deflection, the deflection period, and the energy necessary to produce a deflection shows that, in many cases, a value for none of the intrinsic constants other than the resistance can be chosen arbitrarily, except within limits.

for the inertia constant K, the restoring constant U, and the dynamic constant G, shows that the values for these constants are complex (contain a real and imaginary part) unless

$$D \leqq \frac{T_d^2}{4\pi^2 S_e^2 (R+R_g)} \tag{44}$$

In any design possible of construction, therefore, D must have a value less than this maximum which depends upon the values desired for the operation constants and the value chosen for the resistance R_g, which, however, has no definite upper limit. The values for R_g may, therefore, be chosen entirely arbitrarily, while the values for D must be chosen less than a certain maximum. Values for others of the intrinsic constants, instead of these two, may, within limits, be chosen arbitrarily, but the relations[9] obtainable from equations (25), (26), and (28) which give most promise of being of use in the design of galvanometers for use in class B measurements are

$$D \leqq \frac{T_d^2 m}{4\pi^2 S_e^2 R} \tag{45}$$

$$K = \frac{T_d^3 mn}{4\pi^3 S_e^2 R} \tag{46}$$

$$U = \frac{T_d mn}{\pi S_e^2 R} \tag{47}$$

$$G = \frac{T_d n}{\pi S_e} \tag{48}$$

where

$$m = R/(R+R_g) \tag{49}$$

and

$$n = \tfrac{1}{2}\left[1 \pm \sqrt{1 - 4\pi^2 D S_e^2 R / T_d^2 m}\right] \tag{50}$$

The $\pm$ sign in equation (50) shows that even after values are chosen for R_g and D there still remains a choice between two sets of values for K, U, and G. It will be seen, too, that if D is small in comparison with its maximum possible values, each value in one set is much smaller than the corresponding value in the other set.

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[9] Equations somewhat similar to (45) to (48) were published by Jaeger (Annalen der Physik, 326, p. 76, 1906) and discussed briefly with reference to changes in a particular galvanometer necessary for a certain change in the period and total critical resistance. Later Diesselhorst (Zeitschrift für Instrumentenkunde, 31, p. 250, 1911) used the same equations for finding values for the inertia constant, the damping constant, the restoring constant, and the dynamic constant necessary to give chosen values for the sensitivity, the period, the total critical resistance, and a factor depending upon the logarithmic damping on open circuit. Neither of these authors separates the resistance of the galvanometer from its total critical resistance. That is, they do not make a complete distinction between those of the operation constants with which the user of the galvanometer is concerned and the intrinsic constants, and, as the equations are stated, it would seem that an unnecessary condition is imposed on account of the way the factor depending upon the logarithmic decrement on open circuit enters. In reality no unnecessary condition is imposed, since the user of a galvanometer does not care what the decrement on open circuit is and, consequently, the designer may choose any value for the factor which seems easily realized in the construction.

If the smaller values are chosen, the construction may be much more difficult and the completed instrument may be much more delicate than is necessary, considering the values for its operation constants and the values chosen for its resistance and damping constant. Galvanometers in which more than half of the critical damping is caused by the current which flows in the main winding, as a result of the voltage generated by the moving system, have the larger values for K, U, and G; others have the smaller values. From the standpoint of the design there is a decided difference, depending on whether the + or the − sign, which occurs here and later, is used, and sometimes it will be desirable to use one and sometimes the other.

In the design of galvanometers for use in class A measurements the relations obtainable from equations (25), (26), and (30), which give most promise of being of value, are

$$D \leqq \frac{T_d^2 Rm}{4\pi^2 S_i^2} \tag{51}$$

$$K = \frac{T_d^3 Rmp}{4\pi^3 S_i^2} \tag{52}$$

$$U = \frac{T_d Rmp}{\pi S_i^2} \tag{53}$$

$$G = \frac{T_d Rp}{\pi S_i} \tag{54}$$

where

$$m = R/(R+R_g) \tag{55}$$

and

$$p = \tfrac{1}{2}\left[1 \pm \sqrt{1 - 4\pi^2 S_i^2 D / T_d^2 Rm}\right] \tag{56}$$

Here, in case T^2Rm is very large in comparison with $4\pi^2 S_i^2 D$ and the negative sign of equation (56) is used, expansion of the radical shows that p may be considered equal to $\pi^2 S_i^2 D/T_d^2 Rm$. This value of p substituted in equations (52), (53), and (54) gives

$$K = \frac{DT_d}{4\pi} \tag{57}$$

$$U = \frac{\pi D}{T_d} \tag{58}$$

$$G = \frac{\pi D S_i}{T_d} \tag{59}$$

These equations [10] may also be obtained directly from the relations

$$D = 2\sqrt{KU}, \quad T_d = 2\pi\sqrt{K/U} \text{ and } S_i = G/U \tag{60}$$

[10] In equations (59) and (75) we have considered that $m=1$, which we may do, since R is large in comparison with R_g or there is a condenser in series in the circuit.

which apply in case the instrument is critically damped with the main galvanometer circuit open.

In the same way, it follows from equations (25), (27), and (31) that the relations most likely to be of service in the design of galvanometers for use in class D measurements are

$$D \leqq \frac{m}{\epsilon^2 S_n^2 R} \tag{61}$$

$$K = \frac{2T_b mj}{\epsilon^2 S_n^2 R} \tag{62}$$

$$U = \frac{2mj}{\epsilon^2 T_b S_n^2 R} \tag{63}$$

$$G = \frac{2j}{\epsilon S_n} \tag{64}$$

where

$$m = R/(R + R_g) \tag{65}$$

and

$$j = \tfrac{1}{2}\left[1 \pm \sqrt{1 - \epsilon^2 D S_n^2 R/m}\right] \tag{66}$$

Finally from equations (25), (27), and (33) it follows that the relations which give most promise of being of service in the design of galvanometers for use in class C measurements are

$$D \leqq \frac{Rm}{\epsilon^2 S_q^2} \tag{67}$$

$$K = \frac{2T_b Rml}{\epsilon^2 S_q^2} \tag{68}$$

$$U = \frac{2Rml}{\epsilon^2 S_q^2 T_b} \tag{69}$$

$$G = \frac{2Rl}{\epsilon S_q} \tag{70}$$

where

$$m = R/(R + R_g) \tag{71}$$

and

$$l = \tfrac{1}{2}\left[1 \pm \sqrt{1 - \epsilon^2 D S_q^2/Rm}\right] \tag{72}$$

In case Rm is large in comparison with $\epsilon^2 S_q^2 D$ and we use the negative sign of equation (72), expansion of the radical shows that we may consider $l = \epsilon^2 S_q^2 D/4Rm$. This value of l substituted in equations (68), (69), and (70) gives

$$K = \frac{T_b D}{2} \tag{73}$$

$$U = \frac{D}{2T_b} \tag{74}$$

$$G = \frac{\epsilon S_q D}{2} \tag{75}$$

Equations (57), (58), (59), (73), (74), and (75) apply only in those cases in which critical damping is brought about under conditions in which the voltage generated by the motion of the moving system has no appreciable effect upon the magnitude of the current in the main winding. If the damping is supplied mainly by a current induced in an auxiliary winding closed upon itself, and if g and r' are its dynamic constant and resistance, and D' is the damping constant with both main and auxiliary winding open, the constants of the auxiliary winding must be such that

$$\frac{g^2}{r'}=2\sqrt{KU}-D' \tag{76}$$

If the galvanometer is to be used in a circuit whose resistance, $R'+R_g$ including that of the galvanometer, is not excessively high, then for critical damping it is necessary that

$$\frac{g^2}{r'}=2\sqrt{KU}-D'-\frac{G^2}{R'+R_g} \tag{77}$$

It should be noted that the operation constants all appear on the left-hand side of the equations, and also that those of equations numbered (45) to (50) pertain to class B measurements, those of equations numbered (51) to (59) pertain to class A measurements, those of equations numbered (61) to (66) pertain to class D measurements, and those of equations numbered (67) to (75) pertain to class C measurements.

These equations show the maximum value the damping constant of a galvanometer can have and have chosen values for its resistance, external critical resistance, period, and sensitivity. They also show what values the inertia constant, the restoring constant, and the dynamic constant must have in order that a galvanometer may have chosen values for its resistance, damping constant, external critical resistance (and in some cases the resistance of the apparatus with which the galvanometer is to be used), and the particular period and sensitivity with which we may be concerned. The equations may, therefore, be used in the general design of galvanometers.

3. PROCEDURE IN THE GENERAL DESIGN

Since a value for the resistance of the galvanometer may be chosen arbitrarily and any value taken for its damping constant, less than a certain maximum shown by the first of each set of equations, much is left to the judgment of the person making the general design, and the difficulties encountered in the detail

design and construction and the performance of the galvanometer, in matters other than the values of its operation constants, will depend in no small degree upon the judgment used.

In carrying out a general design the first thing to do is to choose a value for the resistance of the galvanometer, and if one is entirely at a loss in the matter, he may take it equal to that of the rest of the circuit in which the galvanometer is to be used, that is, take R_g equal to R. This makes m, which occurs in most of the equations, equal to one-half. Using this value for m in the first of the set of equations pertaining to the class of work in which it is intended that the galvanometer shall be used, gives the maximum value D can have. Taking D equal to half this maximum value and using the + sign in the last equation of the set makes n (or l or p or j) equal to 0.85. Using these values for m and n the corresponding values for K, U, and G can readily be calculated. This gives us a set of values for the intrinsic constants such as will give the specified values for the operation constants.

While it is not to be presumed that this particular set of values will lend itself most readily to the detail design and construction, it may be used in making a preliminary detail design. A little consideration of the detail design will show, in most cases, that *the resistance of the galvanometer can, to advantage, be made much less than the external critical resistance*, and in a moving coil type of galvanometer, of low external critical resistance, the larger part of the resistance may be in the suspensions rather than in the coil. Ordinarily, such a preliminary detail design will enable us to choose revised values for R_g and for D, such that with the corresponding values for K, U, and G they constitute a set of values for the intrinsic constants which are more easily realized in the detail design and construction.

To get the best results in the general design of a galvanometer, one should be reasonably familiar with the performance and construction of somewhat similar galvanometers (know both their operation and their intrinsic constants) and know fairly definitely the properties of the materials available for the construction of the proposed galvanometer. Usually the general design should be considered as subject to slight modifications until the size of the mirror, the size of the wire, number of turns of and dimensions of the winding, the strength of the magnetic field, the kind of material, section and length of the suspensions, and most of the details of construction are decided upon.

4. DESIGN FOR CURRENT SENSITIVITY

Here the problem is to design a galvanometer which shall be suitable for use in class A measurements, that is, in those measurements in which the galvanometer serves to detect or measure a small current in a circuit of high resistance. (See p. 212.) In general, this resistance may be presumed to be so high that no account need be taken of the effect of the resistance of the galvanometer, or of the voltage generated by the motion of its moving system upon the magnitude of the current. Stating the situation in a slightly different way, the power which can reasonably be dissipated in the resistance of the galvanometer ($R_g i^2$) plus that which can be converted into mechanical power ($i\ e'$, where e' is is the voltage generated by the motion of the moving system) is so small in comparison with the power supplied to the circuit that no account need be taken of it. We may, therefore, make the resistance, or any other of the intrinsic constants, practically as large as we please.

More specifically, the problem is to design a galvanometer to be critically damped in a circuit of very high resistance and have a certain deflection period and current sensitivity.

Equations (51) to (56), inclusive, give the relations which must be satisfied in the general design. Here, R is the resistance which, when connected in series with the galvanometer, gives critical damping. With the galvanometer connected in a circuit of very high resistance, it is the value of the shunt necessary to produce critical damping. If there is a second winding for which the ratio of dynamic constant to resistance is such as will, in itself, give critical damping, then the shunt may be dispensed with, that is, R may be taken as indefinite. There is, therefore, no definite limit for the value of R. It is desirable, however, either to have a shunt of only moderately high resistance or to dispense with it entirely. If R is to be only moderately high, the first matter to be decided upon in the general design is the resistance of the galvanometer. In this choice much will depend upon the size and properties (especially magnetic impurities and thickness of the insulation) of the wire available for winding the coil; the size and kind of wire available for the suspensions; the period, sensitivity, size of mirror, and ruggedness of the instrument desired; and the skill of the person who is to construct it. If the sensitivity is to be high the winding should be of fine wire, the number of turns should be fairly large, and the magnetic field should be strong. However, there is little or no advantage in increasing the number

of turns to the point at which the inertia constant increases as rapidly as the dynamic constant.

Having decided upon a value for the resistance of the galvanometer, experience shows that the external critical resistance may be chosen from 2 to 20 times R_g, the resistance of the galvanometer (10 to 100 times R_g if the deflection period is long). Next, a value may be chosen for the damping constant D. This value must necessarily be less than the maximum corresponding to the chosen values for the resistances and specified values for the deflection period and current sensitivity as shown by equation (51). The corresponding values for the inertia constant K, the restoring constant U, and the dynamic constant G may then readily be calculated from equations (52) to (56), inclusive, using the positive sign in equation (56). The preliminary values for the intrinsic constants, found in this way, may be modified to suit better the conditions met in the detail design.

As an example, assume that a galvanometer is desired to have a deflection period of six seconds, a current sensitivity of 2000 mm per microampere and be critically damped with a resistance in parallel of 2000 ohms; that is, a galvanometer for which

$$T_d = 6,\ S_i = 2000,\ \text{and}\ R = 2000$$

Here the sensitivity and resistance are expressed in the units commonly used, while up to this point it has been assumed that all quantities would be expressed in cgs units or in the same system of units.

Expressing the current sensitivity in terms of the deflection on a scale 1 meter in front of the mirror in millimeters per microampere, the resistances in ohms, the deflection period in seconds, and the intrinsic constants, other than the resistance of the galvanometer, in cgs units, requires a change in the constant in the equations expressing the relations between the various constants. Making this change in equations (51) to (56), which give the relations which must be satisfied in this case, they may be written as follows:

$$D \leqq \frac{T_d^2 Rm}{S_i^2} \tag{78}$$

$$K = \frac{0.32 T_d^3 Rmp}{S_i^2} \tag{79}$$

$$U = \frac{12.7 T_d Rmp}{S_i^2} \tag{80}$$

$$G = \frac{64\,000 T_d Rp}{S_i} \tag{81}$$

where

$$m = R/(R+R_g) \tag{82}$$

and

$$p = \tfrac{1}{2}\left[1 \pm \sqrt{1 - S_i D/TRm}\right] \tag{83}$$

From experience with other galvanometers, we would judge that it would be convenient to make the coil and suspensions in such a way as to have a resistance in the neighborhood of 200 ohms. Using this value for R_g and 2000 for R, it follows from equation (82) that

$$m = 0.91$$

and using this value for m and the specified values for the operation constants, it follows from equation (78) that

$$D \leqq 0.016$$

Taking

$$D = 0.008$$

gives, from equation (83),

$$p = 0.85$$

Values for K, U, and G are then obtained from equations (79), (80), and (81), using these values for m and p and the specified values for the operation constants. Proceeding in this way gives

$$K = 0.027,\ D = 0.008,\ U = 0.029$$

$$G = 330\,000, \text{ and } R_g = 200$$

These constitute a set of values for the intrinsic constants which give the specified values for the operation constants.

These values for the period, current sensitivity, and external critical resistance are those given by the Leeds & Northrup Co.[11] for their type of high sensitivity galvanometer. The value chosen for the resistance, however, is considerably less than the value which they give. Consequently, their values for the other intrinsic constants must differ slightly from these values.

It is to be understood that these values for the intrinsic constants would, in all probability, be changed slightly to better satisfy conditions met in the detail design. However, a change of a hundred ohms in the resistance of the galvanometer, so that we could use a particular size of wire that might be available, would not necessitate any very appreciable changes in the other intrinsic constants. If, on considering the detail design, it should be found that it would be better to have all the intrinsic constants larger, all that would be necessary would be to choose a correspondingly larger value for R.

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[11] See Leeds and Northrup, Bulletin No. 228.

In designing a galvanometer to be critically damped without a shunt, the simpler equations (57), (58), and (59) may be used. Changing the constant of the latter to correspond with the unit of sensitivity used here, these may be rewritten as follows

$$K = 0.080 T_d D \tag{84}$$

$$U = 3.14 D / T_d \tag{85}$$

$$G = 15700 S_i D / T_d \tag{86}$$

It will be noticed that the resistance of the galvanometer does not appear here and that there are only three conditions to be satisfied. Therefore, in addition to the resistance any other one of the intrinsic constants may be chosen entirely arbitrarily. The problem of the design, however, is not materially different from that just considered, except that some means must be provided for producing the necessary damping, since conditions are such that the voltage generated by the motion of the moving system can have no appreciable effect upon the magnitude of the current in the winding. If this is to be accomplished by an auxiliary winding closed upon itself, the constants of this winding may be readily calculated after K and U are determined, providing D', the damping constant with both main and auxiliary windings open, is small in comparison with $\sqrt{KU}$ or is known approximately. From equation (76) it follows that the relation which must be satisfied is

$$\frac{g^2}{r'} = \left[2\sqrt{UK} - D'\right] \times 10^9 \tag{87}$$

where g is the dynamic constant of the auxiliary winding in cgs units and r' its resistance in ohms.

In case one does not care to work out the details of a design to such an extent as will give specified values for the operation constants and yet wishes to construct a galvanometer having a high current sensitivity and a short deflection period, he should make the ratio of the value for the dynamic constant to the value for the inretia constant [12] as large as practicable. This follows from equations (84) and (86), which shows that for this case

$$\frac{S_i}{T_d^2} = \frac{51 G}{K} \times 10^{-7} \tag{88}$$

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[12] Mather, Philosophical Magazine, 29, p. 434; 1890.

5. DESIGN FOR VOLTAGE SENSITIVITY

Here the problem is to design a galvanometer which shall be suitable for use in class B measurements, that is, in those measurements in which the galvanometer serves to detect or measure a small voltage in a circuit of fairly low resistance. (See p. 213.) The resistance of the galvanometer and, during the time the deflection is changing, the voltage generated by the motion of its moving system have a marked effect upon the magnitude of the current, and consequently upon the magnitude of the torque acting upon the moving system.

Usually the apparatus with which the galvanometer is used (and in which, under definite conditions, there is a voltage to be detected or measured) has a certain resistance between the terminals to which the galvanometer is connected. If this resistance is R and the voltage e, the maximum current that may be drawn from these terminals is e/R and the power which may be dissipated in apparatus connected between the terminals can not under any condition be more than $e^2/4R$. This, therefore, is the maximum possible value for power available for producing a deflection of the galvanometer.

However, except by a motion of the moving system, this electrical power can not be converted into mechanical power. Therefore, the amount of energy available for producing a deflection of the galvanometer is $T_d e^2/4R$. The potential energy represented by the deflection must necessarily be considerably less than this amount, since the displacement of the moving system takes place according to a particular law, as shown by equation (18), so that the back or generated voltage is not, except at certain instances, of the magnitude necessary for galvanometer to receive the maximum available power. Further, some power is lost in the resistance of the winding and in air friction, and in the auxiliary closed winding, if there is one.

The problem, therefore, is very different from that which has just been considered. Instead of having all the power one might care to dissipate in the resistance of the galvanometer and use in producing a deflection, the amount of power available is definitely limited. Consequently all possible ways in which the power is used, such as in the resistance of the winding, in the damping frame or auxiliary winding, in case there is to be one, as well as in air friction and in twisting the suspensions or turning the magnet,

it must considered. It might, therefore be better to speak of the power sensitivity, or possibly the energy sensitivity, rather than the voltage sensitivity. More specifically, the problem is the design of a galvanometer to be critically damped when connected to an external circuit having a resistance R, to have a certain sensitivity S_e to voltage in this circuit, and to have a certain deflection period T_d.

Equations (45) to (50) give the relations which must be satisfied. If the sensitivity is expressed as the deflection on a scale 1 meter in front of the mirror caused by an impressed voltage of 1 microvolt, the resistances are expressed in ohms and the other quantities are expressed in cgs units, these equations may be written as follows:

$$D \leq \frac{T_d^2 m}{S_e^2 R} \tag{89}$$

$$K = \frac{0.32 T_d^3 mn}{S_e^2 R} \tag{90}$$

$$U = \frac{12.7 T_d mn}{S_e^2 R} \tag{91}$$

$$G = \frac{64\,000 T_d n}{S_e} \tag{92}$$

where

$$m = R/(R_g + R) \tag{93}$$

and

$$n = \tfrac{1}{2}\left[1 \pm \sqrt{1 - DRS_e^2/T_d^2 m}\right] \tag{94}$$

The first thing to be done is to decide upon a value for R_g, the resistance of the galvanometer. This value when substituted in equation (93) gives a value of m. Then from equation (89) the maximum possible value for D, the damping constant, is obtained. Choosing a smaller value for D, a value for n is obtained from equation (94). These values for m and n, and the specified values for the operation constants substituted in equations (90), (91), and (92) give values for the remaining intrinsic constants K, U, and G.

It will be observed that the higher the sensitivity and the shorter the deflection period desired, and the higher the resistance of the apparatus with which the galvanometer is to be used, and also the higher the resistance of the galvanometer is made, the smaller[13] D, K, U, and G must be. For galvanometers of high sensitivity and short period the difficulties met in the detail design and construction become greater the smaller we attempt to make the

[13] The value of G, however, is but little affected by the value of R, of R_g, or of T_d.

inertia and restoring constants. In the general design of such galvanometers we should therefore aim to keep these constants about as high as is possible. Since particular values usually are desired for T_d, R, and S_e the only place where there can be much choice is in the values of m and n, which should be kept as near unity as is practicable. That is, the values for R_g and D should be so chosen that R_g is small in comparison with R, D is small in comparison with its maximum possible value, and the plus sign in equation (94) should be used. The procedure given here is one that we have been using for the past three years, and a fairly large number of general designs have been made. In a few cases the corresponding detail designs have been worked out and the galvanometers constructed.

For example, let us consider the general design of a moving coil galvanometer to be critically damped with an external resistance of 20 ohms, to have a deflection period of 10 seconds, and a sensitivity of 20 mm per microvolt, that is, have

$$R=20,\ T_d=10,\ \text{and}\ S_e=20$$

From experience in the construction of sensitive galvanometers we know that for the suspensions of this galvanometer very fine wire must be used, and that it would be difficult to make their resistance much less than 10 ohms. We also know that there need be no difficulty in making the coil so as to have a resistance of less than 5 ohms. Taking $R_g=12$ gives, from equation (93), $m=0.62$, and this value of m gives, from equation (94), $D\leqq 0.008$. From experience in the construction of somewhat similar galvanometers we know, if we are to use an approximately uniform radial field with a coil 8 to 10 mm wide and 10 to 15 mm long, we would have difficulty in making the damping constant much less than 0.004. Taking this value for D gives, from equation (94), $n=0.85$. Values for the remaining intrinsic constants are then obtained from equations (90), (91), and (92) by substituting these values for m and n and the specified values for the operation constants. This procedure gives as a complete set of values

$$K=0.021,\ D=0.004,\ U=0.0084$$

$$G=19\ 500,\ \text{and}\ R=12$$

The galvanometers [14] which we designate as type M have (with certain adjustments) very nearly these values for their intrinsic constants.

[14] Wenner, Weibel, and Weaver, Physical Review, 3, p. 497; 1914.

It may be of interest to notice that the maximum values possible for the different intrinsic constants are as follows:

$$K = 0.040,\ D = 0.008,\ U = 0.016$$

$$G = 23\ 000, \text{ and } R_g \text{ no limit}$$

and that K, U, and G can be made to have these values only in case R_g can be made very small in comparison with R, and D can be made very small in comparison with its maximum possible value. It may also be of interest to notice that should a galvanometer be desired having the same external critical resistance and sensitivity but a deflection period only half as long, the inertia constant would necessarily be only one-eighth as large.[15] Also the suspension or restoring constant, instead of being larger, would necessarily be only about one-half as large.

6. DESIGN FOR $\int idt$ SENSITIVITY

Here the problem is to design a galvanometer suitable for use in class C measurements, that is, in those measurements in which the galvanometer serves to measure a small quantity or current impulse. (See p. 213.) Usually the current impulse results from condenser charge or discharge, so that while resistance in the circuit affects the duration, in most cases, it does not affect the magnitude of the impulse.

Only where the resistance is excessively high, much higher than there is any need for making the winding of a galvanometer, is there any appreciable reduction in the throw because of the resistance. The problem of the general design is therefore somewhat similar to that in which the sensitivity to be considered is to current, except that the relation between the ballistic period and the intrinsic constants is very different from the relation between the deflection period and the intrinsic constants. More specifically, the problem is to design a galvanometer to have certain particular values for its ballistic period and $\int idt$ or quantity sensitivity, and be critically damped on open circuit or when shunted with a fairly high resistance.

For example, consider the design of a galvanometer to be critically damped on open circuit, to have a ballistic period of two seconds and an $\int idt$ sensitivity of 1000 mm per microcoulomb. The problem, then, is to design a galvanometer for which

$$T_b = 2,\ R = \infty,\ \text{and } S_q = 1000.$$

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[15] For a quick-acting sensitive galvanometer the advantage of making the inertia constant much smaller than has previously been customary is pointed out by Moll, Proceedings of the Koninkijke Akademie van Wetenschappen te Amsterdam, 16, p. 149; 1913.

Let us assume that we will make the resistance any value which the detailed design shows will be convenient, and that we will make $K = 0.10$ cgs units. Since the $\int idt$ sensitivity is expressed as the deflection on a scale 1 meter in front of the mirror in millimeters per microcoulomb, if the other quantities are to be expressed in cgs units, then equations (73), (74), and (75), which apply in this case, may be written as follows:

$$D = 2K/T_b \tag{95}$$

$$U = K/T_b^2 \tag{96}$$

$$G = 13\,600\, S_q K T_b \tag{97}$$

From these the values of D, U, and G may be readily obtained by a substitution of the particular values of K, T_b, and S_q. This gives for the intrinsic constants

$$D = 0.10,\ K = 0.10,\ U = 0.025$$

$$G = 860\,000, \text{ and } R_g = \text{any value}$$

If the galvanometer is to be of the moving coil type and have the pole pieces and core so shaped as to give practically a radial field, we know from experience that the damping constant D' (resulting mainly from air friction) is likely to be in the neighborhood of 0.03 unless some effort is made to make it unusually small. For critical damping the difference between D and D', amounting to 0.07, must be provided for in some way. If this is to be by an auxiliary winding closed upon itself, then from equation (76) the relation between its dynamic constant g expressed in cgs units and its resistance r' expressed in ohms must be such that

$$\frac{g^2}{r'} \times 10^{-9} = 0.07$$

If conditions are met in the detail design which show that there would be an advantage in having larger or smaller values for the intrinsic constants, there would be no difficulty in finding a new set differing widely from these yet giving the same values for the operation constants.

For a galvanometer to be shunted so that critical damping is brought about by current induced in the main winding instead of an auxiliary winding, the problem of the general design is not materially different. A value must be chosen both for the resistance of the galvanometer and for the resistance of the shunt. These, together with the values for the operation constants, give

from equations (71) and (67) an upper limit for the damping constant. Choosing a smaller value for the damping constant, values for the remaining intrinsic constants may be calculated from equations (68), (69), and (70). Here the constants of the equations must be changed unless all qauntities are expressed in cgs units.

7. DESIGN FOR $\int edt$ SENSITIVITY

Here the problem is to design a galvanometer which shall be suitable for use in class D measurements, that is, in those measurements in which the galvanometer serves to detect or measure small voltage impulses in circuits of fairly low resistance. (See p. 213.)

Both the resistance of the galvanometer and the voltage generated by the motion of the moving system have a marked effect upon the magnitude of the current (the latter, of course, only while the deflection is changing) and consequently upon the magnitude of the ballistic throw. The problem of the design is therefore somewhat similar to that in which the sensitivity to be considered is to voltage. The effects of the resistance of the winding, of the damping on open circuit, and of the voltage generated by the motion of the moving system are very similar. More specifically, the problem is to design a galvanometer which shall be critically damped when connected to an apparatus having a certain resistance, have a certain sensitivity to a voltage impulse in the apparatus, and have a certain ballistic period.

The relations which must be satisfied are given by equations (61) to (66), inclusive. In case the sensitivity is expressed as the deflection, on a scale 1 meter in front of the mirror, in millimeters per microvolt-second, the resistances are expressed in ohms and the other quantities are expressed in cgs units, these relations may be written as follows:

$$D \leqq \frac{5.4m}{S_n^2 R} \tag{98}$$

$$K = \frac{10.8 T_b m j}{S_n^2 R} \tag{99}$$

$$U = \frac{10.8 m j}{T_b S_n^2 R} \tag{100}$$

$$G = \frac{147\,000 j}{S_n} \tag{101}$$

$$m = R/(R + R_g) \tag{102}$$

$$j = \tfrac{1}{2}\left[1 \pm \sqrt{1 - DRS_n^2/5.4m}\right] \tag{103}$$

In carrying out any particular design the first thing to be decided upon is the resistance of the galvanometer, and it will be observed that, in general, the higher the value chosen for the resistance the smaller all the other intrinsic constants must be made. For sensitive galvanometers, difficulties are met in the detailed design and construction if the inertia and restoring constants must be small. It is therefore desirable to choose a value for R_g which, if practicable, is small in comparison with R. Having decided upon a value for R_g, a value must be chosen for the damping constant, D. If practicable, this value should be not more than half the maximum possible value as given by equation (98). A much larger value for D would require making K and U considerably smaller.

For example, let us consider the general design of a galvanometer to be critically damped in a circuit of 25 ohms, not including the resistance of the galvanometer; to have a ballistic period of 1 second, and a sensitivity of 1 mm per microvolt-second. That is a galvanometer for which

$$R = 25,\ T_b = 1,\ \text{and}\ S_n = 1.$$

If the galvanometer is to be of the moving coil type, from experience in the construction of similar galvanometers we know that it would be difficult to make the resistance much less than 10 ohms. Taking R_g as 10 gives, from equation (102), $m = 0.71$. This value m gives, from equation (98), 0.038 as the maximum possible value for D. Taking $D = 0.019$ gives, from equation (103), $j = 0.85$. Values for the remaining intrinsic constants are then readily obtained from equations (99), (100), and (101), and we have as the complete set

$$K = 0.062,\ D = 0.019,\ U = 0.062$$

$$G = 62\,600,\ \text{and}\ R_g = 10.$$

With suitable material there should be little or no difficulty in carrying out the detailed design and construction so as to obtain very nearly these intrinsic constants. A consideration of the detailed design would, no doubt, suggest some slight changes in the general design.

8. DETAIL DESIGN

The detail design consists in deciding upon the number of turns and shape of the winding, the size and kind of wire to be used, the strength of the magnetic field, or the size, shape, etc., of the magnet; the length, size, shape, and kind of material to be used for the suspensions, the size and shape of the mirror, all so as to

obtain for the intrinsic constants the values which the general design shows they should have. It also includes devising some of the procedures to be followed in the construction, means for adjusting, a consideration of the general appearance, etc. In case the galvanometer is to be of the moving magnet type, means for astaticizing and shielding against outside magnetic fields, arrangement of soft iron cores, if any are to be used, etc., must also be decided upon.

If materials were available having just the form we should like and having just the properties we should like, the detail design, and also the construction of sensitive galvanometers, would present no special difficulties. However, the materials available are generally not in the most suitable form and are more or less lacking in the properties we should like them to have. The winding, which should be nonmagnetic, is usually slightly magnetic in fairly weak fields and more magnetic in strong fields, the wire available for the winding and for the suspensions has resistance, the magnet steel may not be capable of maintaining the desired field unless the magnet is made larger or the air gap smaller than we should like, the wire of the winding must be insulated and the insulation adds to the mass and takes up valuable space, and even the air damps the motion of the moving system and thus adds to the energy necessary to produce a deflection. If the galvanometer must be especially delicate, much will necessarily depend upon the skill and even the temperament of the person who is to carry out the construction. For these reasons many compromises must be made so that the design may not be so much what is desired as what seems most feasible under the particular circumstances. Such compromises as are made must necessarily be made in such a way as to retain those values for the intrinsic constants given by the general design, except as the general design may be modified to meet better the conditions encountered in the detail design or construction.

Because the materials available may not be in a suitable form or may not have suitable properties, the detail design and construction may present serious difficulties or even be impossible of realization. It is in the avoiding or overcoming of these difficulties that real skill in galvanometer design and construction is shown.

However, the detail design of galvanometers or of any particular galvanometer is not a matter which could properly be considered fully in this paper. In a previous paper [14] a brief detail descrip-

tion is given of a particular type of instrument, and possibly in later papers matters pertaining to detail design and construction may be considered.

IV. SUMMARY

1. Attention is called to the fact that galvanometers are used critically damped or approximately critically damped in four distinct classes of measurements, in each of which the sensitivity with which the user is concerned is with respect to a different quantity. In the first, it is with respect to current in a circuit of high resistance; in the second, it is to voltage in a circuit of fairly low resistance; in the third, it is to current impulse in a circuit containing a condenser; and in the fourth, it is to voltage impulse in a circuit of fairly low resistance.

2. A clear distinction is made between these sensitivities and other operation constants in which the user of the galvanometer may be interested, and the inertia constant, the damping constant, the restoring constant, the dynamic constant, and the resistance, these latter being the construction or intrinsic constants with which the maker of the galvanometer is concerned.

3. It is pointed out that for any one of these classes of work the user is concerned at most with but three operation constants, while the galvanometer has five intrinsic constants. Hence, different values for the intrinsic constants may give identical values for the operation constants; or, in the design of a galvanometer to have particular values for its operation constants, values for some of the intrinsic constants may be chosen arbitrarily.

4. It is shown that in all cases the value for the resistance of a galvanometer may be chosen arbitrarily without interfering in any way with the determination of values for the remaining intrinsic constants such that the galvanometer will have the previously selected or specified values for the set of operation constants pertaining to any one of the four classes of measurements. However, in many cases, unless a fairly low value is chosen, there may be difficulties in carrying out the construction.

5. For galvanometers to be used in the detection or measurement of current in a circuit of high resistance or of current impulse in a circuit containing a condenser, it is shown that in addition to a value for the resistance a value for any other one of the intrinsic constants may also be chosen arbitrarily; and values may be calculated for the other three such as will give previously selected values for the operation constants pertaining to either of these two classes of measurements. This is in case the galvanom-

eter is to be critically damped with the winding open-circuited. In case a shunt is to be used to bring about critical damping, corresponding arbitrary choices in values may be made.

6. For galvanometers to be used in the detection or measurement of voltage or voltage impulse in circuits of fairly low resistance, it is shown that when a value is chosen for the resistance an upper limit for the damping constant may at once be calculated. Choosing a value under this limit, values for the remaining intrinsic constants may readily be calculated to give previously selected or specified values for the operation constants pertaining to either of these two classes of measurements. Attention is called to the fact that the higher the sensitivity and external critical resistance, and the shorter the period desired, the smaller must be the inertia constant and the restoring constant.

7. The detail design of a galvanometer, which consists in deciding upon the size and proportions of the magnet; the size, shape, and number of turns and size of wire for the winding; the size, shape, and kind of material to be used for the suspension, and all such matters, are considered briefly; and it is pointed out that much must necessarily depend upon the properties of the materials available for use in the construction and, especially in case the galvanometer must be delicate, much must also depend upon the skill in manipulation of the person who is to carry out the construction.

This paper relates to a part of a general investigation of galvanometers which is being carried on by the author and Ernest Weibel, who has made valuable suggestions concerning matter discussed here.

Washington, December 30, 1915.